Multilingual Students Working with Illustrated Mathematical Word Problems as Social Praxis

Laura Caligari, Eva Norén, and Paola Valero

Abstract Word problems in mathematics can present challenges for multilingual students. Previous research shows that the language and cultural contexts of word problems are major obstacles for many students. This study advances the research by considering the work with illustrated word problems as a social praxis. We specifically ask what social, cultural and linguistic experiences multilingual students mobilise and create when working with illustrated word problems. Data were collected from eight multilingual students in fourth grade. The students were given word problems to solve individually. The data were qualitatively analysed based on a four-fold structure of social praxis as a framework. Findings reveal that word problems will always mobilise exclusion. The study concludes that when working with illustrated word problems teachers need to recognise this condition and balance the ways of working with word problems. Policies need not only to promote linguistic and cultural diversity explicitly. They could open real possibilities for policy enactment where the linguistic and cultural differences embedded in illustrated mathematical word problems are discussed and negotiated.

Keywords Illustrated mathematical word problems · Multilingual students · Social praxis

L. Caligari (✉) · E. Norén · P. Valero
Department of Mathematics and Science Education, Stockholm University, Stockholm, Sweden
e-mail: laura.caligari@mnd.su.se

E. Norén
e-mail: eva.noren@mnd.su.se

P. Valero
e-mail: paola.valero@mnd.su.se

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1 Introduction

Educational policy in Sweden has attended to research results that, since the 1990s, show the benefit of including and supporting students' mother tongue as a resource for instruction and learning in mathematics (Barwell, 2009; Moschkovich, 2007; Prediger & Schueler-Meyer, 2017). According to the Swedish national curriculum, all teaching must be adapted to each student's prerequisites and needs. Teaching should promote students' continued learning and knowledge development based on their backgrounds, past experiences, languages and knowledge (Skolverket, 2011). Indeed, all students with first languages other than Swedish have the right to tuition in their first language. "Mother tongue" is a subject in the Swedish curriculum, thus, students living in municipalities that have the resources and organise for the subject will have the chance of studying their first language in school. Studies in Swedish mathematics classrooms (Norén, 2010; Ryan, 2019) show that multilingual students benefit when they are able to utilise their linguistic resources and cultural identities.

However, for multilingual students and in multilingual classrooms—where students speak at least two languages and where more than one language could be used in the classroom (Barwell, 2009, 2018)—the fact that Swedish is the instructional language poses challenges. This results in the systematic underperformance of multilingual students in mathematics compared to their first-language Swedish peers (Skolverket, 2019a, 2019b). One of such challenges is how students' linguistic and cultural knowledge and experience are significantly taken as a resource. In fact, the last quality review by the School Inspectorate revealed a lack of knowledge about multilingual students' cultural backgrounds and experiences in many schools (Skolinspektionen, 2010). Furthermore, research has shown that there is a lack of understanding of multilingual students' experiences in multilingual mathematics classrooms and how they can become significant for mathematics (Svensson Källberg, 2018). In other words, the Swedish educational policy has a clear intention of inclusion but keeps on producing a clear exclusion of multilingual students, particularly in mathematics. This is problematic, in terms of equity, for students learning mathematics in a second language (Barwell et al., 2019).

In this chapter, we explore this challenge as it unfolds in one important element of school mathematics: the reading and solving of illustrated mathematical word problems. In the Swedish mathematics curriculum, problem-solving is a privileged competence that students must develop (Skolverket, 2011). It is emphasised that problems should relate to students' everyday lives (Skolverket, 2017) as a way to bridge school mathematics and out of school reality. Despite their centrality in many curricula, it is known that word problems are difficult for many students, in particular for multilingual ones (Barwell, 2009; Cooper & Dunne, 2000). If word problems are defined as "one way to express beliefs about how everyday experience and mathematics should be related in order for math learning to take place effectively" (Lave, 1992, p. 75), the connections between the beliefs about everyday experiences encapsulated in word problems and the actual experiences of students are a clear predicament for research and practice. Existing research has shown that, when

solving word problems about everyday life, multilingual students tend to draw on their first language (Planas & Civil, 2013) and on their cultural or religious identities (Barwell, 2009) to increase their understanding and learning. But to maintain and continue developing, it is important that the experiences they create and draw on are valued and utilised. This is hardly the case.

This chapter advances the research by considering the work with illustrated word problems as a social praxis, in which word problems and students' experiences intertwine as "learning and knowledge [become] situated in social interaction" (Gutiérrez, 2013, p. 45). The overall aim of this study is to explore what experiences multilingual students draw on and create when solving illustrated mathematical word problems and to discuss the implications for mathematics classrooms. This study more specifically addresses the question of what social, cultural and linguistic experiences do multilingual students mobilise and create when engaging in the social praxis of solving illustrated mathematical word problems in Swedish. Based on our findings, we provide some insights for policy.

2 Mathematical Word Problems and Multilingual Students

Mathematical word problems have existed at least since Babylonian times, as a pedagogical tool to induce mathematical activity. Working with mathematical word problems is nowadays regarded as an effective way for students to learn and become successful in mathematics (Boonen et al., 2016; Dyrvold et al., 2015). Word problems combine the cues of mathematics within a context in which the mathematics is to be used. Thus, they often reflect general characteristics of a given society (Barwell, 2018; Gerofsky, 1996).

Research shows that students' engagement with the contexts in word problems is challenging. Some aspects of the context are often ignored in students' solutions, and seemingly "realistic" items may cause confusion and misunderstanding, leading to nonsensical answers (De Corte et al., 2000; Greer, 1993, 1997). It can be challenging for native speakers and multilingual students to communicate mathematical ideas and concepts in relation to word problems (Barwell, 2009; Clarkson, 2007). This has raised the question of whether word problems may be too artificial or the contexts too unfamiliar for students. Explorations of such difficulties have revealed that students engage in considering the contexts in word problems. Multilingual students often relate to their own cultural experiences and home cultures when solving and constructing word problems of their own (Barwell, 2009). Barwell (2018) points that students' solutions only become nonsensical when the solutions lack students' interactive processes and the sense-making that led up to them. He stresses that word problems must be understood as "socially constructed, deployed and interpreted texts" (p. 102). They are social texts, and as such reflect a tension between normative academic practice and students' life experiences (Barwell, 2018).

As an important element of a normative academic practice, mathematical word problems constitute a genre of mathematical literacy, with specific linguistic, structural and contextual features that students need to familiarise themselves with (Barwell, 2009; Gerofsky, 1996, 2004). Students' knowledge of the strategies and routines in the mathematical literacy genre are therefore significant. The word problem genre has certain textual forms but can also be regarded as social (Barwell, 2018). Students need to understand the norm—how they are expected to read, interpret, respond—so it reflects their mathematical knowledge. Students from minority language/cultural groups and students from working-class backgrounds often perform less well on word problems than their counterparts (Cooper & Dunne, 2000). In Barwell's (2018) words, word problems can create social stratification between those who are "at home" with the genre and those who are not. Moreover, reading a mathematical word problem in a second language often entails putting more effort into decoding the text and understanding the context than interpreting the mathematical content (Clarkson, 2009). Thus, performance on word problems is partly related to students' knowledge, while stratification of students into groups happens according to their backgrounds.

3 Mathematical Word Problems and Illustrations

Word problems may also include illustrations. Word problems can have a textual and a graphic component, and in some cases these components are more or less related and significant for engaging with the mathematical task. Illustrations are pedagogically intended to help learners visualise the context and thus solve the word problems more easily (Dewolf et al., 2015). Dewolf et al. (2015) explain that the idea with representational illustrations is to help students construct a rich mental model of the mathematical situation and prevent them from only searching for a standard computation of the word problem. Teledahl and Olsson (2021) suggest that students engage written and illustrated word problems in various ways, as an illustration and a written text in a problem can be viewed as two different sources of information and can be treated as isolated, connected or combined. These studies have shown that, despite the good pedagogical intentions and the different types of engagement, students tend to neglect the representational illustrations when solving the problems.

The question emerges, of whether this is also the case when multilingual students meet illustrated word problems. Besides facilitating a cognitive demand, illustrations can be seen as a way of generating a familiar connection between the students and the mathematical task. Research on textbooks and instructional materials points to the fact that mathematical contents are presented together with national cultural elements such as forms of behaviour, artefacts, geography, identities and history (Fan et al., 2018). Doğan and Haser (2014), in a study of Turkish textbooks, have pointed to the potential effects of exclusion produced when students from cultural minorities meet problems that emphasise a particular national identity. Souza and da Silva (2018) have also shown how the use of toys in the problems and illustrations in Brazilian

primary mathematics textbooks offer a traditional gender role for girls to relate to. In other words, word problems in curricular materials incorporate elements of the national culture and have effects on how students from particular groups engage them.

In short, multilingual students mobilise their knowledge and experiences of the world out of school when meeting word problems with their texts and illustrations. The text and illustrations in word problems carry with them particular cultural elements. Then one can expect many issues emerging in the interactions between students and illustrated word problems; and such issues go beyond the linguistic and cognitive capabilities required to successfully engage with the problems. To explore this issue, we propose to move the conceptualizations of illustrated word problems as social texts—as proposed by Barwell (2018)—and cognitive devices—as proposed by Dewolf et al. (2015)—into the terrain of social praxis.

4 Working with Illustrated Word Problems as Social Praxis

Radford's cultural theory of learning (2008a, 2008b, 2018) allows us to understand the work of students solving illustrated word problems as a social praxis, that is, a process where social and cultural forms of knowing are constituted. Such work can be conceived not as an interaction between two independent entities—the student and the problem—but as an entanglement between the student and the illustrated word problems as cultural artefacts, where learning and becoming emerges. The activity of working with problems binds in inseparable ways who the student is—her experiences and ways of making sense mathematically—and the cultural significations that are encapsulated in the illustrated word problem. In this way, the engagement of the student with the problem is not just the mediation of her thinking for the purpose of objectifying the mathematical knowledge in the problem, but also a meeting with the cultural significations that are part of the contexts to which the words and illustrations in the problem refer to.

Illustrated word problems instantiate ideas of society, and embody cultural perceptions about mathematics, the world and individuals. The cultural environment provides illustrated word problems with "raw material" and regulates what is being perceived as norm in a society, reinforcing cultural perceptions about the world and individuals (Radford, 2018). Thus, word problems understood as artefacts facilitate the assimilation of perceptions not only of mathematics—their technical aspect—but also of the very same culture they encapsulate—their cultural aspect. This is what Radford highlights with the idea that artefacts are given meaning with respect to cultural systems of signification (e.g. Radford, 2018, p. 454).

Students are expected to identify with situations in the problems and understand them since they are supposed to be part of the culture shaping illustrations and word problems in curricular materials. However, multilingual students' subjective meanings may clash with the cultural objective meaning in illustrated word problems, since their culture, language and experiences may differ from those of the dominant

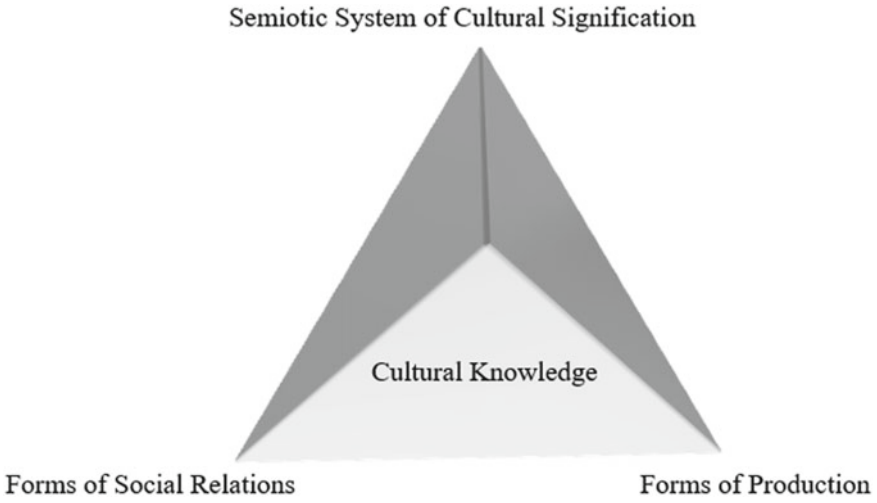

Fig. 1 The four-fold structure of social praxis (Radford & Empey, 2007, p. 235)

culture encapsulated in problems in textbooks for school instruction. Radford and Empey's (2007) four-fold structure of social praxis (see Fig. 1) provides a framework to examine multilingual students' work with illustrated word problems and explore how their experiences are mobilised in that social praxis.

The first element is *Forms of Social Relations,* such as interactions, or divisions of labour. In this study, this element implies the interaction between the student and the illustrated word problem as well as statements about previously experienced interactions when working with illustrated word problems. Division of labour refers to assumed or assigned roles, which are often more like characters, that differentiates the self and the other. We explore the roles that students have experienced previously working with word problems.

The second element is *Forms of Production*—the artefacts that mediate cultural perceptions like signs and objects, in this study texts and illustrations. Students' interactions with the texts and illustrations can help us explore their experiences, articulated in an interactive process when working with the problems.

The third element is the *Semiotic System of Cultural Signification*, for instance, a cultural perception about the world and individuals. Students expressed perceptions that have a normative function, such as what is good, right, the truth, methods of inquiry or the legitimate forms of knowledge representation relating to working with illustrated word problems. This element also helps us explore how students perceive themselves, experience culture and inherent perceptions.

The fourth element is *Cultural Knowledge*, which is an epistemological point of view, and involves the knowledge available in a culture. This refers to a process that includes the three former elements. In this study, the fourth element relates to unspoken rules relating to activity, language and norms students reveal when working with illustrated mathematical word problems.

5 Methodology

This qualitative study explores the entanglement between multilingual students and illustrated word problems as cultural artefacts, as they work with them. Our focus is not on students' right or wrong solutions to the problems, but on their experiences.

5.1 About the Participating Students

We asked a school in a socio-economically disadvantaged area outside a major Swedish city if some of their students wanted to participate in our study. We chose to do our study in fourth grade because in Sweden the transition from third- to fourth-grade entails moving from lower to middle school, changing teachers, engaging new mathematics textbooks and more reading. We also expected that students would have experienced working with illustrated world problems due to their centrality in the curriculum. We informed the fourth graders' parents about the study and ethical issues, and those interested signed a consent form. Eight multilingual students agreed to participate. After their mother tongue, Swedish is their second language and English a third language since the latter is compulsory at least from third grade. The eight students had all started attending this school in first grade. In their class of 25 students, all had Swedish as their second language and 12 different mother tongues were represented in the classroom. Swedish was the instructional language.

On the day of the interviews, students were informed individually about the study and the procedure, and they gave verbal consent before starting. The interviews and engagement with the word problems were conducted individually by the first author in a room next to the students' classrooms. No time limit was imposed, and each encounter lasted about 20 min. All students' names are pseudonyms.

To begin with, the students got to answer some interview questions, like, "*what do you learn when working with word problems?*", "*would you prefer to have word problems in your first language?*" and "*how do you work with word problems in class?*" Some questions needed follow-ups, like, "*explain a little more about […]*" when the student had answered too briefly. The aim with the questions was to get background information about the students, talk about their experiences and thoughts related to word problems, and help them feel comfortable with the interview situation. We did not ask questions about their mathematics scores, mathematics abilities or language proficiency; we wanted to focus on their experiences. Even if we noticed that when students spoke Swedish, they seemed fluent, we had no idea of their level of vocabulary and competence in Swedish.

After the interview questions, the students were given the word problems, blank paper, a pencil and an eraser. We asked the students to read the word problem out loud and to think aloud while working. When they fell silent, they were asked, "*what are you thinking now?*". We did not specify whether they should think aloud in their

164. How much older is a) Daddy than mummy? b) Jonas than Miranda?
165. Look at the map. The journey started in Ljungby. The first day they went to Jönköping and then to Mjölby. How far was it?

164. Hur mycket äldre är a) pappa än mamma? b) Jonas än Miranda?
165.Titta på kartan. Resan började I Ljungby. Första dagen åkte de till Jönköping och sedan till Mjölby. Hur lång *sträcka* blev det?

Fig. 2 This is the family Svensson from Ljungby in Småland. Last summer they went to Kolmården zoo (Undvall et al., 2011, p. 43; illustration by Unenge [Johan Unenge has given his permission to use his illustrations in this text.])

first or second language, but to proceed as they normally would when solving a word problem.

5.2 About the Tasks

The word problems were chosen based on our knowledge that these problems had troubled fourth-grade multilingual students at another school (Norén & Caligari, 2021). The word problems were selected from a year four mathematics textbook (Undvall et al., 2011) that was not used in the participants' school, so they were not familiar with the book or the problems. The problems include informational illustrations (Elia & Philippou, 2004) containing numbers not provided in the text but essential for solving the problem. For example, the illustrations include prices of items, ages of people and distances.

Two of the tasks had the theme *Kolmården* (see Fig. 2). Kolmården is a huge zoo with animals from around the world. Two other tasks had the theme *The Market* (see Fig. 6).

5.3 About the Analysis

The interviews were audio recorded, stored and transcribed. The data consisted of transcripts and participants' solutions produced during the interviews. The analysis began with a close reading of the transcripts, which were then searched for illustrative passages relating to the categories in the analytical framework. We analysed the students' statements and the work processes and sense-making that lead them to their solutions, and finally highlighted social, cultural and linguistic experiences. Table 1 indicates the kind of connections that we established between the framework

Table 1 Analytical framework

Category	Sub-category	Sub-category	Identification rule(s)
Forms of social relations	Interactions Roles	• teacher • student • word problem • themselves • others	Expressed interactions between the student and teacher, student and other students, student and word problem Expressed roles students give themselves and/or others
Forms of production	Artefacts	• text and illustration	How the students read and produce texts and illustrations
Semiotic system of cultural signification	Perceptions	• ideas • methods • oneself	Expressed perceptions relating to word problems. For example, good, bad, difficult. Expressed perceptions of method when working on word problems Expressed perceptions of oneself
Cultural knowledge	Unspoken rules	• activity • language • norms	Expressed demonstrated understanding of unspoken rules when working with word problems, words and illustrations

and evidence emerging in the interviews and conversations around the work with the problems.

6 Findings

6.1 Forms of Social Relations

In the interviews, the participants said that on occasion, when a word problem is too difficult, they work in smaller groups in an adjoining room and that "*the teacher explains difficult words*". But usually, they worked individually with word problems in the classroom. One participant explained that it should be "*quiet so you don't disturb each other*". The participants' interactions with the word problems were evident while they read and looked at the task, and some students said things like, "[*d*]*o I need to count this?*" or "[*w*]*ait, what?*" or "[*i*]*s this a task*?" And sometimes while writing a solution, "[*h*]*ow do I write this?*" The interaction seemed to be with the word problem, or with themselves.

One of the roles highlighted by students was the teacher–student role. The student's role implies learning and the teacher is assumed to be the only one who knows which words to practice and learn; in other words, "if we don't understand we can ask the teacher for tasks to practice at home".

The idea of the other is related to contrast and evaluation. The other is a fellow student to compare oneself with. Either the other is defined as the one having difficulties when working with word problems or the one who thinks they are easy: "*They* [word problems] *may be too difficult for some, but I think they are fun*", or the opposite: "*The others think they* [word problems] *are easy, but I don't*".

6.2 Forms of Production

As the participants read the word problems out loud it appeared clear that some students switched their readings between the text and illustration, and some of the words seemed to be difficult to read and understand. The participants got stuck and it showed as they slowed down their reading, sounded the words or reread them. "*This is the family Svensson from Lj… Lju… Ljung… by in Små… land… in Småland. Last… summer, they went to Kolo… Kolmården… zoo. How much older is… Peter forty-two years, Anna thirty-nine years, … Mar… Miranda three years, O… Olivia ten years, Jonis twelve years. How much older is daddy than mummy, Jonis than… Miranda?*" (see Fig. 2). The most challenging words to read were frequently (perhaps unfamiliar) names of cities such as *Ljungby*, *Småland* and *Jönköping* and names of people such as *Patrik*, *Miranda, Olivia* and *Jonas.* One participant reasoned "*Miranda? Miranda is a drink in Morocco*". While three participants read "[How much older is] *Jonis than Miranda*" instead of Jonas, as written in the word problem. Jonis is a common Arabic name.

This challenge while reading is something that needs to be taken into consideration as it can affect students when they are working on word problems with time constraints.

In task 165 (see Fig. 2) the illustration of the map carries meaning; that is, it provides information critical to the word problem. Not understanding the map is thus a disadvantage. Likewise, misreading or not understanding how to interpret and respond to a question, like, "how far was it?", can result in the student not demonstrating the required mathematical knowledge. As one participant reasoned out loud while writing an answer: "*How long was the line? It was long! They walked all this way to Kolmården zoo*" (see Fig. 3).

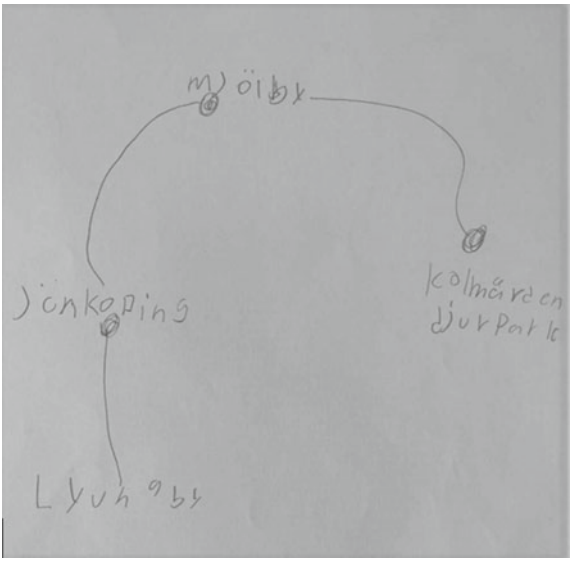

Fig. 3 Student C solution to task165 (see Fig. 2)

6.3 Semiotic System of Cultural Signification

The students said that the main idea of working with word problems is to "… *learn how to count in different ways*" or "*it teaches me different difficult things, like multiplication and division*". Also, there was the perception that word problems are more complicated to work with than mathematical tasks without words. During the interviews, students said word problems were more difficult. If the word problem was considered difficult (by the teacher or the students), they usually read the text with the teacher and focused on difficult words—"*it's good because you get to have more explanations*".

The participants said that when working with word problems the most suitable method is reading and understanding what to do next. They did not refer to illustrations in their statements: "*it is difficult when it is a text, you have to read carefully*"; "*In math, it is a little harder to figure out the text. The hard thing is that you have to understand the text. I don't really understand the text all the time*"; "*you have to read and then you must count*". Before writing their solutions, students sometimes ask themselves "*what should I write?*". The students here are seeking legitimate forms of knowledge representation, which shows that they understand they must produce a text of some kind, also showing familiarity with the word problem genre (Barwell, 2009). Since the students were talking aloud while working with the problems, their interactive process and the meaning making that led to a solution were evident. This revealed, among other things, that some students first verbalised an answer before writing it down. In Task 164a (see Fig. 2), participants A, E and D counted aloud while raising a finger at each accentuated word, then looked at the fingers and said an answer; that is, they said, "*thirty-nine,* ***forty***, ***forty-one***, ***forty-two****… three*", three

being their answer as they held three fingers in the air. Despite the fact they had used the same method verbally, their written accounts differed (see Fig. 4). Participant A did not finish writing a solution when noticing that it did not add up as expected. E wrote an addition that matched the pre-calculated result and D wrote a word answer to the question. Even participant B counted aloud raising a finger on each accentuated word, and got another result: "***thirty-nine***, ***forty***, ***forty-one***, ***forty-two***… *four*"; the written addition then matched what had been done while counting out loud (see Fig. 5).

Other statements about what is considered a legitimate method when working were: "*you need time to think*" and "*it needs to be quiet*"—the later a social praxis made visible in the classroom that reinforces the idea of difficulty.

Students also expressed a perception that practising skills lead to improvement. One student said: "*I have to confess; I don't know how to solve this; I find it really difficult because I don't know how to do it. So, I don't know how to count it, should I do plus or minus? I don't know, but I'm not going to be ashamed of it; I know I have a little trouble with this, and I should practice*". This perception also highlights self-awareness as a person responsible for one's own knowledge acquisition and feelings about mathematics. Other students' declarations, showing the same self-awareness, are often connected to statements of evaluation, like "*I'm not good at math*", "*I don't like math so it's not fun. If you can [do math], it's more fun*" or "[*y*]*ou have to practice to become smart*".

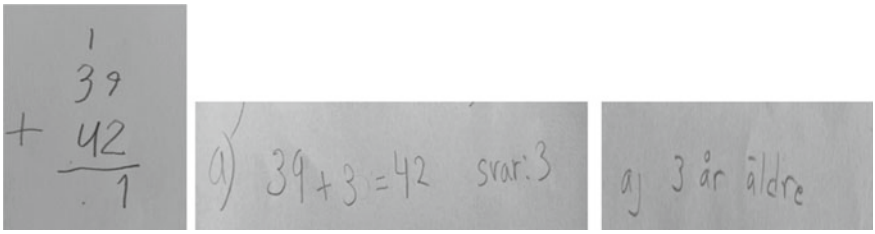

Fig. 4 Solutions to task 164a (see Fig. 2). Reading from left to right: Student A [incomplete], Student E ["answer: 3"] and Student D ["3 years older"]

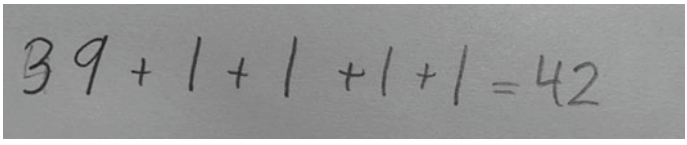

Fig. 5 Student B solution to task 164a (see Fig. 2)

6.4 Cultural Knowledge

Cultural knowledge can be represented in the cultural idea of how you are meant to demonstrate the ability to reason and apply mathematics in real contexts (see Figs. 4 and 5). Or as we could see in Task 165 (see Figs. 2 and 3) even the activity of reading a map can be involved when working with a word problem. It was also shown that mathematical words like distance [*sträcka*] on a map caused trouble because it was read as *streck* [line], a word that has a similar pronunciation and almost the same spelling. Moreover, lines were drawn on the map to show distances (see Figs. 2 and 3) and the word *sträcka* in Swedish also means stretch and reach, so students had to figure out what the word meant.

When students solved Task 736 (see Fig. 6) we discovered something else. Participant B read the word problem out loud, looked at the illustration, said, "*I can't do it*" and did not explain why. Two other participants also reacted to this problem, especially the everyday words in the question. The words *serietidningar* (comic books) and *tidningar* (magazines) appeared in the word problem but the illustration used the word *serier* (series).

Student F said aloud, *Peter bought 16 magazines, where are the magazines? Is it magaz… no those are books… series, where are the magazines? Is it any magazines? Here it says series; is it the same thing? It can't be a series you watch on TV, or…*?

While student A reasoned that "*there [were] different words in the same task!? Can a series be magazines?*", the three different words all referred to the same comic books, thus confusing the students.

In Swedish, *serietidningar*, *tidningar* and *serier* can refer to the same kind of publication. But in many other languages, it would be different. In English, for example, comic books and magazines are not the same kind of publication. Even though the word was not a mathematical term, it took effort and time for the participants to figure out how to solve the task.

One unspoken norm and activity is that the students processed the word problems in the language of instruction. They all interacted with the word problems in Swedish. In the interviews, five of the students said they would not like to have word problems in their first language because they found Swedish easier to read and understand,

735. Christina bought 14 oz of candy. How much did she have to pay?
736. At one stand comic books are sold. Peter bought 16 magazines. How much did he have to pay?

735. Christina köpte 4 hg godis. Hur mycket fick hon betala?
736. I ett stånd såldes gamla serietidningar. Peter köpte 16 tidningar. Hur mycket fick han betala?

Fig. 6 The market scene, with the word *serier* up to the left (Undvall, et al., 2011), words underlined by us

while three participants replied that it would be nice to have both. Of those three, two clarified that it was not because they did not understand it in Swedish, but for the opportunity to practise their first language. One of them said it would be nice because it would probably be easier to understand than Swedish. The students also said that word problems require that they understand what they read and a knowledge of which calculation methods to use. They claimed that "*you practice both mathematics and Swedish*".

7 Discussion

7.1 The Social

Working with an illustrated word problem is a social experience, and the students' work with the tasks showed how they drew on their former experiences of reading and working with word problems. When they interacted with both the text and the illustrations, they posed more questions to the text but retrieved information from both, showing they were aware of illustrations as a source that could be connected to or combined with the text (Teledahl & Olsson, 2021). But there were some specific situations that interrupted their work, as when a question could not be answered, an illustration was neglected, or a calculation did not add up as intended. Students' experiences related to their (un)familiarity with the procedures of working with mathematical word problems. So, even if the student's work showed they had met word problems earlier, they also showed a gap in their knowledge of the conditions required when working with illustrated word problems. The social activity needed when working with illustrated word problems is a mixture of normative academic practice and life experiences (Barwell, 2018).

The students explained that they wanted to show their thoughts mathematically in writing when working with word problems but were not always sure how to do that. This challenge to communicate mathematical ideas and concepts confirms what Barwell (2009) and Clarkson (2007) found in their studies. Students said that interacting in smaller groups and having the teacher explaining difficult words enabled better understanding. So, they can benefit from collaboration. Students are familiar with posing questions, and they have experienced it as a good way to learn. This means that students could be given more opportunities to talk to each other about the tasks and together clarify words and illustrations in the word problems they are working on.

When not knowing which arithmetic to use for solving a word problem most students tended to say they were not good at math; they took on a specific role, the role of the student who is weak in mathematics. However, some of those students also said they felt responsible for practising their own skills or cultivating their own learning; it could indicate that the student rejects a certain role imposed on those who cannot solve a mathematical problem (Gutiérrez, 2013).

7.2 The Cultural

Students' statements reveal that they perceive a classroom culture where working individually is the established way—the social praxis—of working with word problems in their classroom. We did not do classroom observations; our statements are based on the student's perceptions. And they are expressing a cultural idea and a norm (Barwell, 2018) of how to work with word problems, namely, that it is good to work individually because it makes it easier to think. And, therefore, the need of silence in the classroom seems obvious to them. However, their experience is that they need to speak to make the illustrated word problems understandable, and it becomes noticeable when the words are perceived as difficult, as well as when the context in an illustration is unfamiliar.

It could be that when the students are familiar with the cultural context encapsulated in the illustrated word problems, they have the preconditions needed for working, and then silence becomes natural.

The above findings affirm that word problems always mobilise exclusion. The struggles with solving word problems and unfamiliarity with contexts is what Cooper and Dunne (2000) wrote about years ago. Students' backgrounds play a key role and solving mathematical word problems can sometimes be a test of who the students are. Our participants were not familiar with Kolmården, a typical place for middle class (Swedish) parents to take their children to in summer. We believe the unfamiliarity with the context shows the importance of knowing about students' earlier experiences so the right support can be given to them when solving illustrated word problems. In the case of Kolmården, the context can be explained, and the students could be encouraged to talk about their experiences of zoos.

7.3 The Linguistic

Although everyday words can be troublesome and it takes time to figure out their meaning, most of the students took the time to work through this process. However, this meant that less time was spent on the mathematical work demanded by the word problem, which is consistent with Clarkson's (2009) findings. The students' previous experiences of everyday and mathematical words had an impact on the way they understood and took on the work. The students showed that they were able to draw on experience to work out the meaning of a word, for example, through the process of elimination, working out that, given the context, the word *serier* could not refer to TV series. But drawing on own experiences can be hard when tasks are connected to specific Swedish cultural phenomena (e.g. people, cities, places) that students are not familiar with. The social praxis is interrupted, and a gap appears within the norms. A single word, like a homonym that represents different concepts, then causes confusion, like the Swedish word *sträcka,* which may not be part of students' everyday vocabulary. It can therefore be tricky to know whether students

are struggling with the mathematical words, or their second language, or whether they do not understand a concept. As Schleppegrell (2007) states, there are always linguistic challenges when learning mathematics in a second language.

The students also said they found word problems complicated and trickier than mathematical tasks without words. Their statements reveal that it is because they find some words in the problems confusing and it takes time and effort to figure them out. It could mean that the students would benefit from connecting to their first languages. Employing students' mother tongues could adapt teaching to better suit students' needs, to promote their knowledge development based on past experiences and language as recommended by the Swedish curriculum (Skolverket, 2011).

The student said that word problems helped them practice and improve both mathematics and Swedish. And it is not surprising that students find Swedish easier because Swedish has always been their language of instruction and they have always had Swedish textbooks. Furthermore, none of the eight students had attended school elsewhere. The social expectation is that they should be able to solve word problems in Swedish. It is a normative expectation relating to Swedish classroom culture (Norén, 2015) and a sign of their adaptation to the norm of using "only" Swedish in the classroom.

8 Concluding Remarks and Recommendations for Policy

By using the four-fold structure of social praxis as a theoretical framework, we were able to identify the experiences multilingual students mobilised and created when solving illustrated mathematical word problems in Swedish.

Understanding the work with illustrated word problems as social praxis showed that multilingual students experiences related to school culture, having solved word problems before, and remembering what teachers have told them. They tended to rely on previous experiences of solving word problems even when encountering difficulties. The students seemed to skim over illustrations and sometimes even the context because they were so focused on solving the task. They tried to use their mathematical skills. However, neither their experience with Swedish nor their social experiences seemed sufficient. A precondition of working with word problems could also be to address the cultural experiences of multilingual students more thoroughly. If the latter are acknowledged in an everyday school context, they should be part of the learning activities.

We conclude that multilingual students doing mathematics in a language which is not their first language need more opportunities to work with others and communicate verbally, instead of being asked to work individually and quietly. When students ask spontaneous questions to texts, it is possible to take advantage of this. Students must be allowed to mathematise texts and illustrations collaboratively before working individually. Students might also benefit from being allowed to use their first languages (not simply translations) in problem-solving situations, since much of their life experience is related to their daily lives when they are using their first languages (Barwell,

2009; Planas & Civil, 2013). Students need rich opportunities to enhance their experiences of how to solve illustrated word problems. Students themselves refer to the understanding of "how" to solve and "which" arithmetic to use. Active use of language/s is one way to make sure everyone is involved and shares their experience with problems and how to solve them. In this way teachers can adapt their support to the needs of the students, since it becomes easy to capture students' cultural experiences in the classroom's social praxis, and to build new experiences.

The students seemed to exhibit a strong desire to follow normative social praxis relating to "doing" the arithmetic rigorously and to using a method that they believed the teacher would approve of. To increase students' opportunities to experience everyday words in their second language, we suggest combining different school subjects and working thematically. A theme such as Kolmården, for example, could fit particularly well with Geography, where students could elaborate on meaning making, and thus relate words to maps of Sweden and other countries. This theme could also be linked to Orienteering[1] in sports where students get to learn common words, and the experience of Orienteering, in real life. One could also integrate the subject Swedish into Mathematics, using illustrations to elaborate on different words, concepts and homonyms, and examine the word problem genre in relation to other genres which the students encounter in Swedish. The aim would be to explore ways to visualise and make natural connections among mathematical experiences in everyday life by integrating different school subjects.

In line with the Swedish curriculum, there are opportunities for teachers to develop cooperation with mother-tongue-speaking teachers and mentors. Support from these teachers could help students reflect on mathematical issues in more than one language (Swedish). Moreover, access to one's mother tongue can highlight everyday experiences that are created in students' homes, where they speak their mother tongue. Mother tongue also relates to students' cultural backgrounds and to their parents' practices of learning and doing mathematics (Civil et al., 2005).

Our conclusion is that, as multilingual students participate in the social praxis of solving illustrated word problems, they increase their success in relating their cultural experiences to the cultural contexts in the problems. Support for students should include working explicitly with language in different ways, and making the cultural elements embedded in the word problems visible. Students need to submerge themselves in illustrations, words and contexts, and experience word problems in various ways. By dissecting illustrated word problems, talking about them and looking at how the specific word problems are constructed students can better understand how to go about working with them.

Finally, policies that explicitly work with inclusion of multilingual students by supporting language learning—such as the current Swedish educational policy—need to develop a nuanced view of how language and culture are entangled in the particularities of school practices such as the work with illustrated word problems in mathematics. Explicit inclusive policy formulations are not enough. This may

[1]In Sweden a common "competitive sport in which runners have to find their way across rough country with the aid of a map and compass" (https://www.lexico.com/definition/orienteering).

mean that policy could also promote concrete spaces for teachers' *policy enactment*[2] where language, culture and experiences can be discussed and negotiated in the classroom. In the concrete case of illustrated word problems in mathematics, such spaces of policy enactment may mean that it is not sufficient with allowing children to strengthen their mother tongue to understand the problem with the hope that they can produce a mathematically adequate answer. The issue is rather to open the opportunity of recognising the particular cultural norms and life forms that are embodied in the problems—and the curriculum as a whole—and to bring them in contrast and discussion with respect to other possible life experiences emanating from the diversities of students' lives. These spaces of enactment are indispensable, since it is only in the details of praxis that possibilities for real inclusion can be worked out by teachers in classrooms.

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[2]The notion of policy enactment (Ball et al., 2012) refers to the idea that the most important aspect of a policy is not just the statement of intentions in official documents, but the constant work of translation and recontextualizations of those intentions that teachers do in their everyday work in schools and classrooms.

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Laura Caligari is a Ph.D. student in the Department of Mathematics and Science Education at Stockholm University. Her doctoral research focuses specifically on mathematical word problems and multilingual learners. Laura has worked as a teacher in elementary school for over a decade, mainly with multilingual learners in Sweden.

Eva Norén is Associate Professor at the Department of Mathematics and Science Education, Stockholm University. Her research focuses on multilingual students in mathematics classroom, as well as social, critical and political issues in mathematics education.

Paola Valero is Professor of Mathematics Education at the Department of Mathematics and Science Education, Faculty of Science, Stockholm University, Sweden. Her research interests are mathematics education at all levels; in particular policy enactment processes, curricular development, multiculturalism and multilingualism in mathematics education, and diversity in mathematics teacher education. Currently, her research explores the significance of mathematics and science education as fields where power relations are actualized in producing subjectivities and generating in(ex)clusion of different types of students. She is currently the leader of the Swedish Research Council funded national Graduate school "Relevancing mathematics and Science Education (RelMaS)". Her research has been part of the Nordic Center of Excellence "Justice through education in the Nordic Countries".