Practices in multilingual mathematics classrooms: word problems

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This is a study in multilingual mathematics classrooms where Swedish is the language of instruction. Our aim is to explore what troubles do appear when students work with mathematical word problems and how teachers provide scaffolding for students' learning. Classroom observations and student interviews were conducted. The lessons followed the structure of introducing, modelling, jointly practicing and individually performing. Students' understanding of how to go about in the mathematical word problem genre advanced when they became familiar with the context and worked together by explaining, communicating in pairs and constructing individual word problems.

In this paper, we present a study conducted in multilingual mathematics classrooms where Swedish is the language of instruction. Languages have been seen as resources in multilingual mathematics classrooms since the 1990s (Barwell, 2009; Moschkovich, 2007; Prediger & Schueler-Meyer, 2017). However, few studies in classrooms with multiple languages represented have been pursued. Moreover, multilingual students in Swedish mathematics classrooms still underachieve (Skolverket, 2019). Thus, there is a need of more research in multilingual mathematics classrooms where students' and teachers' linguistic resources are drawn on. Another rational for such research is that multilingual classrooms have become more common in Sweden. Today, numerous mathematics classrooms consist of many mother tongues spoken by students and teachers. These students are all second language learners in mathematics. Research reveal that it can be difficult for students to communicate mathematical ideas and concepts when instruction is in their second language (Barwell, 2009). Furthermore, research) show that word problems are particularly difficult for second language learners (Clarkson, 2007).

Sweden of today is a multilingual country. A little more than 20% of the students in compulsory school use other first languages than Swedish, Arabic being the most common (SvD, 2018). Mathematics teachers are supposed to support second language learners' acquisition in mathematics through content and language integrated teaching (van Eerde & Hajer, 2009; Hajer & Norén,

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2017), meaning that teachers' instruction have to focus on specific language requirements of mathematics. The overall aim for this study is to explore how teachers provide scaffolding when second language learners encounter difficulties when solving mathematical word problems. In this paper we more specifically ask:

1 What difficulties can be discerned when second language learners read and solve mathematical word problems?

2 What strategies do teachers use to scaffold second language learners' proficiencies in solving word problems?

This study, reports from a project that explored and tried out content and language integrated teaching of mathematics in school year 4 and further in year 5, from October 2017 until December 2018. We present students' classroom work with mathematical word problems and concepts, as well as interviews with students.

Word problems and multilingual classrooms

As noted above, studies in multilingual mathematics classrooms have shown that proficiency in the language of instruction relates to attainment in mathematics, and proficiency in two or more languages makes a difference for students' attainment. Nevertheless, strong proficiency in a language that is not the language of instruction has also been shown to have an impact for students' mathematical attainment (Clarkson, 2007).

Gerofsky (1996) defined word problems as a certain genre of mathematical literacy, comparing it with other spoken and literary genres. Word problems are often associated to as real-world problems that students can relate to, though they have been criticized to be too artificial (De Corte et al., 2000) even saying that "word problem solving" is disconnected from the real world. For example, in Greer (1993) 13–14-year-old students in Northern Ireland ignored aspects of the real world when answering word problems. One explanation is that students relate to contexts familiar to them. Barwell's focus on word problems and its genre in some of his research in multilingual mathematics classrooms (2009), showed that multilingual students relate to their own cultural experiences and home culture when solving and constructing word problems of their own. Another clarification is that second language learners often draw informally on their mother tongue when solving word problems, in order to increase their learning (Clarkson, 2009; Planas & Civil, 2013).

Theoretical considerations

We acknowledge the sociocultural nature of the resources second language learners and teachers bring to the mathematics classroom (Moschkovich, 2007).

Furthermore, also in line with the sociocultural tradition deriving from Vygotsky, language and content-based integrated teaching of mathematics and language supportive theories are adopted (Gibbons, 2002; Smit, 2013; van Eerde & Hajer, 2009). This tradition is related to scaffolding processes through challenges and support to promote students' autonomy. Smit (2013) writes that scaffolding is "a certain kind of support provided by teachers to help students move forward" (p. 14). Scaffolding is temporary and will gradually be removed as the structure being fostered becomes more solid and more reliable. Scaffolding in multilingual mathematics classrooms means:

– making the mathematical content understandable by putting it in contexts that the students can relate to,
– promoting students' active language use both orally and in writing in the mathematics classroom,
– offering varied and long-term linguistic support.

Drawing on sociocultural theories, the practices in content and language integrated classrooms often adapts scaffolding (Vygotksij, 1999) in a teaching and learning cycle model presented by Gibbons (2002). The cycle was further developed for mathematics teaching and learning by Smit (2013) in a project on second language learners' reasoning about line graphs. The teaching and learning cycle involves a series of four phases in which a specific genre of text required in a school context is introduced, modelled, together practiced and individually performed by the students. According to Gibbons (2002), the idea is that second language learners have to progressively acquire skills in the language of instruction along a continuum from every-day language to more academic language, from spoken to written language and connected by literate spoken language. However, second language learners don't acquire academic language skills through classroom discourse, like first language students often do. Second language learners need scaffolding in relation to the academic language of each school subject, in this paper mathematical language, says Gibbons (2002).

Methodology

The empirical data is ethnographic (Hammersley & Atkinson, 2007) and derives from classroom observations, fieldnotes, audio recordings of classroom interaction, collections of students' materials, and student interviews. In two of the classrooms (Red School) in this study, besides Swedish, there were nine first languages spoken by the students [1]. In the third classroom (Blue School) there were 13 first languages spoken besides Swedish [2]. Students' parents in both schools were informed about the ethical issues and signed consent forms. All names of schools and students are pseudonyms.

The study draws on 14 participant observations in the Red school and the two classes respectively, in which 40 students are learning mathematics. To verify findings about how students perceived mathematical word problems in the Red school we interviewed students in another school, the Blue School, where students solved some of the mathematical word problems that the students in the Red School had solved. Interviews with 8 students were recorded individually. The students solved word problems, answered questions and talked about their experiences of solving word problems. They took on the tasks by reading the word problems and solving them while ”thinking” aloud.

The methodology is interpretative and relays to knowledge building and cultivates research capability through collaborative analysis and critical reflection of students learning and classroom practices (Calder & Murphy, 2018). Regarding difficulties discerned when students solve mathematical word problems, the analysis of students' interaction with teachers and classmates identified themes (Braun & Clark, 2006). The themes were thoroughly linked to the practice in the classrooms. The analysis also showed that teachers' scaffolding strategies, depended on the difficulties experienced by their students.

Participant observations

Participant observations were conducted in the Red school where two teachers started to change their mathematics teaching towards using more content and language integrated methods. So far, their mathematics teaching hadn't helped their students achieve as expected. Word problems in mathematics were the most challenging area for the students to work with, as well as for the teachers to teach. For example, when the students got the assignment to work with a thematic chapter “The Kolmården Zoo” with a lot of word problems from their textbook, the classroom became “chaotic” (teachers' expression). Besides the two teachers there were mother tongue supervising personnel. Thus, students had opportunities to use their first languages alongside Swedish in the mathematics classroom.

Interviews

Students in the Blue school expressed that the word problems they worked with in the classroom were the ones from their mathematics textbook and that they worked individually with them. The student also explained that it should be “quiet so you don't disturb each other” (student quote). They also said that there were some occasions when they worked in small groups, for example, if the word problem was difficult, then they read the text with the teacher and focused difficult words.

During the interviews in the Blue School, the students said that word problems were different from mathematical assignments without words in the mathematics textbook. The word problems required an understanding of their own reading and they had to decide on what calculation methods to use.

Findings

In the thematic analysis of fieldnotes and audio recordings (from classrooms and interviews), three main themes were discerned regarding what seemed to create difficulties for students when solving word problems: ”difficult” words, ”difficult” contexts and ”conceptual understanding”. In various ways, the themes relate to each other. The analysis also displays strategies for how teachers scaffold their second language learners. The strategies were promoting students to: use their first language, ask questions on the word problems presented, mark ”key words” in the word problems and to actively use Swedish orally and in writing. Teachers' scaffolding was sometimes planned ahead, building on earlier lessons, results on diagnostic tests or text material that was going to be used. At other occasions the scaffolding unfolded while teaching, building on interaction in the classroom, for example students' questions and students answers to teachers' questions. The scaffolding was often relating to students understanding of mathematical concepts.

”Difficult” words

A mathematical word can be difficult, but the reason may be that the concept the word represents may not be understood. It is sometimes hard to define if students have troubles with the mathematical words, their second language or if a student has missed the understanding of a concept. There are always linguistic challenges in learning mathematics in a second language (Schleppegrell, 2007).

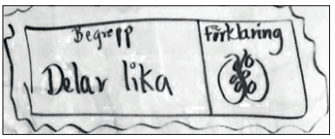
The students in the Red School had regularly failed when solving word problems in the classroom. Therefore, the teachers started systematically to diagnose the students on mathematical word problems One diagnostic test was taken from McIntosh (2008). Firstly, students solved the diagnostic test in Swedish, thereafter, they were offered to solve the same test in their mother tongue. A word problem that was difficult for the students to solve in both languages was:

> Bo cut his apple in half, and then cut one of the halves in half again [*Bo delar sitt äpple i halvor. Sedan delar han ena halvan mitt itu*].
>
> a) How many pieces of apple does he now have?
>
> b) What fraction of the whole apple is one of the small pieces?

The task was formulated in Swedish, though here from the original source in English. The most difficult part of the wording was neither the first part of the sentence, nor the next. However, there was a wording in Swedish that seemed to be difficult for most students, ***mitt itu***[3] (it is not a straightforward translation of ”one of the halves again”). The students had no difficulties when cutting the apple in half, but had to stop reading at *mitt itu*, get an explanation, and then go on cutting one of the halves in half again. Regarding the questions it was obviously a complex word problem to solve. Only four students out of 39 could solve the (b) question in Swedish. One teacher strategy for scaffolding students was that mother tongue speaking teachers explained the words and the

mathematical phenomena to the students on their mother tongue. This was promoted by the Swedish teachers.

When solving the word problem in their mother tongue some students found it was no difference or it was easier, but some students felt more comfortable using their mother tongue. Another scaffolding strategy was to draw and write on the white board and grouping students in smaller groups, giving them follow up tasks to work actively with words earlier articulated in the diagnostic test. One example is shown in figure 1.



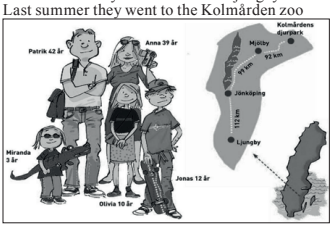
Concept	Explanation
Sharing equally	Drawing of halt an apple

Figure 1. *Photo from the white board*

It was chosen by one teacher because she wanted her students to elaborate more on the Swedish wording *mitt itu*[4] relating it to "share equally in two parts".

When interviewing Blue Schools students, some words in the word problems seemed to be difficult to read and understand. Students got stuck on words and it showed as they slowed down their reading, sounded out the words aloud or reread words. The most difficult words were not always the mathematical words, but names of cities and people. Mathematics words like *sträcka* [distance] on a map caused trouble, because it has the same pronunciation and almost the same spelling as *streck* [line]. Additionally, lines were drawn on the map to show distances (see figure 2). Another example, "there are different words in the same assignment" (student quote) in one of the word problems and the picture that followed the assignment.

This is the family Svensson from Ljungby in Småland. Last summer they went to the Kolmården zoo



164. How much older is a) Daddy than mummy? b) Jonas than Miranda?
165. Look at the map. The journey started in Ljungby. The first day they went to Jönköping and then to Mjölby. How long was it?

164. Hur mycket äldre är a) pappa än mamma? b) Jonas än Miranda?
165.Titta på kartan. Resan började I Ljungby. Första dagen åkte de till Jönköping och sedan till Mjölby. Hur lång *sträcka* blev det?

Figure 2. *Kolmården zoo tasks (Undvall et al., 2011, p. 43)*[5]

In the example in figure 3, two students reacted to the words *serietidningar* [comic books] and *tidningar* [magazines] in the word problem but in the illustration, it said *serier* [series] "is it the same thing? It can't be series you watch on TV?", a student in the Blue school reasoned out loud. The three words all referred to the same comic books, thus, confusing the students[6].

I ett stånd såldes gamla serietidningar. Peter köpte 16 tidningar. Hur mycket fick han betala?

In one stand, old comic books were sold. Peter bought 16 magazines. How much did he have to pay?

Figure 3. *The Market theme, the word* serier *up to the left (Undvall, et al, 2011)* [5]

”Difficult” contexts

In the Red school, the teachers returned to theme about a trip to Kolmården in the textbook. When it was first introduced a month before, students skipped the tasks. Kolmården is a big park with wild animals from different parts of the world. The tasks to solve included a family going there, word problems on the family members' ages and their trip by car to the zoo.

None of the students were familiar with the Kolmården context, thus, a lot of explanations were needed. The scaffolding strategy followed a structure in which one of the teachers started with bringing the first two pictures up on the smart board. The teacher inventoried what the students knew by letting them ask questions and collaboratively communicate about the pictures and it's mathematical content. Teachers wrote on the smart board. Both text and pictures were carrying mathematical meaning. Finally, students solved the word problems and later, in pairs, construed their own word problems for other students to solve. When working in the textbook with other themes, like the Market (figure 3), the context was familiar to most students, which helped them solve the word problems.

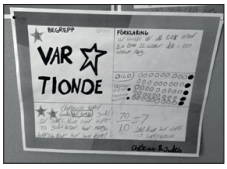
Conceptual understanding

When students worked by themselves in the textbook, in the beginning of school year 4 in the Red School, many of them tried to skip the word problems. Students were saying “the word problems are too difficult” and “I don't know what to do”. When elaborating with the Kolmården theme the analysis showed it was obviously difficult for the students to understand the context and the wording. One example: “Look at the map”. The picture of the map (figure 2) carries meaning to the word problem, thus, not understanding the map is a drawback. The wording ”mitt itu” when cutting an apple, and other examples from word problems like ”every tenth”, serier [comic books] and ”addition”, also created troubles relating to concepts.

Before starting to solve word problems and in line with planned scaffolding, the teachers taught strategies like, “look for difficult words and words you don't understand, mark the numbers, mark the words, underline the question/s, what information can you find”? Teachers and students together defined which words were mathematical concepts and which were some kind of key words or every day words used for constructing word problem. Words to define were

picked from the word problems. Students categorized the words and teachers wrote them in lists on the white board, one for concepts, and one for key words. The teachers often led collaborative classroom talk with students' questions as starting points.

Successively, students were assigned concepts from the list. In pairs they



Concept EVERY TENTH	Explanation Every 10th, for example, after ten flashes, it turns into a different color.
Task Christian buys candy. Every tenth candy is a piece of liquorish. He bought 70 pieces of candy. How many pieces of liquorish did he buy?	Solution $\frac{70}{10} = 7$ Answer: He bought 7 pieces of liquorish

Figure 4. *Student material*

elaborated on the concepts by writing explanations, so that others would understand the concept, while also constructing word problems for others to solve. Figure 4 is an illustration of that certain activities, such as collaborative work, encouraged students to talk to each other and to actively take on various assignments. The regular work in pairs made students exchange mathematical knowing. In activities like the exploration and construction in figure 4 of the concept "every tenth", students got used to examine concepts, explain to each other and communicate. This scaffolding strategy helped students solve and construe word problems of their own. Thus, it became obvious that the scaffolding strategies used, motivated students to solve word problems at the end of the school year 4.

Discussion

This study shows the importance of examining, on the one hand, what difficulties second language students encounter when working with word problems and, on the other hand, how teachers can scaffold and support second language learners' mathematical word problem solving skill, by reducing difficulties observed.

In the Red School classrooms, second language learners worked more engaged when familiar with the context and when they had been given time to work systematically with word problems. For example, the word problems relating to Kolmården, compared to the word problems relating to the Market, created challenges for the second language learning students. One reason could be that students related the Market to their experiences and thus their home culture and mother tongue. The Kolmården theme was the opposite, students couldn't relate to their experiences (Clarkson, 2009; Planas & Civil, 2013). Students in the Blue school also got stuck on certain words when reading word problems. Those words, like *sträcka* and *streck* [distance and line], are usually

learned in relation to everyday language, hence not familiar to all second language learners. In the Red school, students' active use of language was promoted both orally and in writing. For example, when elaborating on mathematical concepts or when construing word problems for their peers. Students were offered linguistic support, mostly in Swedish but also in their mother tongue, scaffolding them to negotiate meaning (Gibbons, 2002). In our analysis we noted that even though the teachers in the Red school were not deliberately adapting the teaching and learning cycle (Gibbons, 2002; Smit, 2013) their lessons followed the structure of introducing, modelling, jointly practicing and individually performing. In other words, our study indicates that second language learners' understanding of how to go about in the mathematical word problem genre advance when they become familiar with the context, get access to their mother tongue and are promoted to work together by explaining, communicating in pairs and construing individual word problems.

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Notes

1 Arabic, Dutch, French, Persian, Polish, Russian, Spanish, Tigrinya and Turkish.

2 Albanian, Arabic, Bengali, Chaldean, English, Greek, Igbo, Kurdish, Moroccan Arabic, Romani, Somali, Turkish, and Persian.

3 The wording *mitt itu* would probably not have caused the same troubles for a native Swedish speaking student because the wording is part of the everyday language, and not the academic language.

4 "Mitt itu" can in Swedish also be to "dela lika" or "dela i två lika delar" *share equally* in two parts.

5 Illustration by Johan Unenge, who has given his permission to use it in this paper.

6 It is complicated to explain this in English, it makes sense in Swedish but it doesn't in English. In English comic books and magazines doesn't mean the same kind of publication, but in Swedish *serietidningar, tidningar* and *serier* can refer to same publication, here, the ones soled at the market.