Eksempel - Uten symmetri egenskaper

Bestem Fourier-rekken til

$$f(x)=\begin{cases}2+x & -2\le x<0\\ 0 & 0\le x<2\end{cases}\quad ;\quad f(x+4)=f(x)$$

Grafen har ingen symmetriegenskaper:

$$a_0=\frac{1}{2\cdot 2}\int_{-2}^{2} f(x)dx=\frac{1}{4}\int_{-2}^{0}(x+2)dx=\frac{1}{4}\left[\frac{1}{2}x^2+2x\right]_{-2}^{0}=\frac{1}{2}$$

$$a_n=\frac{1}{2}\int_{-2}^{2} f(x)\cos\frac{n\pi x}{2}dx=\frac{1}{2}\int_{-2}^{0}(x+2)\cos\frac{n\pi x}{2}dx=\frac{1}{2}\left[\int_{-2}^{0}x\cos\frac{n\pi x}{2}dx+\int_{-2}^{0}2\cos\frac{n\pi x}{2}dx\right]$$

$$=\frac{1}{2}\left[\left[(\frac{2}{n\pi})^2\cos\frac{n\pi x}{2}+x\frac{2}{n\pi}\sin\frac{n\pi x}{2}\right]+2\frac{2}{n\pi}\sin\frac{n\pi x}{2}\right]_{-2}^{0}=\frac{2}{(n\pi)^2}\left(\cos 0-\cos(-n\pi)\right)$$

$$=\underline{\underline{\frac{2}{(n\pi)^2}\left(1-(-1)^n\right)}}=\frac{4}{(2n-1)^2\pi^2},\quad n=1,2,3,\cdots$$

$$\sin(n\pi)=\sin(-n\pi)=0\quad,\cos(n\pi)=\cos(-n\pi)=(-1)^n\ ,\ \frac{1}{\frac{n\pi}{2}}=\frac{2}{n\pi}\ \text{og}\ \frac{1}{\left(\frac{n\pi}{2}\right)^2}=\frac{4}{(n\pi)^2}$$

$$b_n=\frac{1}{2}\int_{-2}^{2} f(x)\sin nxdx=\frac{1}{2}\int_{-2}^{0}(x+2)\sin\frac{n\pi x}{2}dx=\frac{1}{2}\int_{-2}^{0}x\sin\frac{n\pi x}{2}dx+\int_{-2}^{0}2\sin\frac{n\pi x}{2}dx$$

$$=\frac{1}{2}\left[\left[(\frac{2}{n\pi})^2\sin\frac{n\pi x}{2}-x\frac{2}{n\pi}\cos\frac{n\pi x}{2}\right]-2\frac{2}{n\pi}\cos\frac{n\pi x}{2}\right]_{-2}^{0}=\frac{1}{2}\left[-\frac{4}{n\pi}(-1)^n-\frac{4}{n\pi}(1-(-1)^n)\right]=\underline{\underline{-\frac{2}{n\pi}}}$$

Ved å bruke a_0 , a_n og b_n i Fourier-rekken, får vi dermed:

$$\mathrm{Ff}(x)=\frac{1}{2}+\sum_{n=1}^{\infty}\frac{4}{(2n-1)^2\pi^2}\cos\frac{(2n-1)\pi x}{2}-\frac{2}{n\pi}\sin\frac{n\pi x}{2}$$